Prior-Free Bayesian Optimal Double-Clock Auctions

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Bayesian mechanism design:

- Fruitful conceptual framework and analytical tool
- "Fragile" because of its dependence on the fine details of the environment
  - E.g., optimal reserves vary with distributions
- Wilson (1987): "Only by repeated weakening of common knowledge assumptions will the theory approximate reality" as is "required to conduct useful analyses of practical problems"
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Introduction

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This paper

- Bridges the gap between robust and Bayesian mechanism design
- Develops a double-clock auction for a two-sided environment with privately informed buyers and sellers that is:
  - Prior free
  - Endows agents with obviously dominant strategies
  - Preserves the privacy of agents who trade
  - Asymptotically Bayesian optimal (any weights on revenue and efficiency)

Prior-Free Double-Clock Auctions (DCAs)

- Defined without reference to distributions
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Further Properties of Prior-Free DCAs

- Deficit free
- Weakly group strategy-proof
- Operational for any size of market
- Requires only limited commitment by the designer
- Its equilibrium outcome remains an equilibrium outcome in a full-information first-price double auction
- Ex post individually rational

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Further Results

For two-sided setups,

- We characterize the Bayesian optimal mechanisms that preserve the privacy of trading agents
- Show that our mechanism converges to the privacy preserving Bayesian optimum as estimation errors vanish

Moreover, we establish

- As a corollary, the impossibility of ex post efficient privacy preserving trade (when full trade is sometimes but not always optimal)
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Extensions

We can incorporate:

- Real-time diagnostics regarding tradeoffs associated with continuing the DCA
- Revenue thresholds
- Asymmetries among agents
  - Caps on the number of agents of a particular group who can trade
  - Favoritism towards particular groups of agents

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Role for Flexible Two-Sided Exchanges

- Novel environments and one-off reallocations of assets
  - Designer and participants can't rely on past experience
  - Desirable to dispense with Bayesian notions both for the rules of trade and for the equilibrium strategies
  - Obvious dominant strategies aid inexperienced bidders
- More generally:
  - Privacy preservation reduces participation concerns and costs
  - Envy-freeness guards against claims of "arbitrary and capricious" design (esp. if designer is Government)
  - Deficit-freeness protects the designer

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Literature

- Wilson (1987), Bergemann and Morris (2005, 2012)
- Myerson and Satterthwaite (1983), Gresik and Satterthwaite (1989), Williams (1999)
- McAfee (1992), Milgrom and Segal (2015)
- Segal (2003), Baliga and Vohra (2003), Matsushima (2005)
- Goldberg, Hartline, and Wright (2001)

Key features that set our paper apart:

- Prior free for any market size
- Permits an implementation via DCA
- Methodological contribution: Observe and exploit the connection between spacings of order statistics (empirical) and virtual types (theoretical)

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Illustration: Symmetric Setup and Revenue Extraction

- n buyers and m sellers; single-unit demand and supply
- Buyers' values v are independent draws from $F(v)$ on $[\underline{v}, \overline{v}]$ with continuous positive density $f(v)$
- Sellers' costs c are independent draws from $G(c)$ on $[\underline{c}, \overline{c}]$ with continuous positive density $g(c)$
- Everyone is risk neutral and has quasilinear utility
- Regularity holds:

$$\Phi(v) \equiv v - \frac{1 - F(v)}{f(v)} \quad \text{and} \quad \Gamma(c) \equiv c + \frac{G(c)}{g(c)}$$

  are increasing

- Designer and agents do not know the distributions
- Designer knows that regularity holds
- Objective: asymptotic revenue maximization
- Later allow for:
  - objective of weighted sum of revenue and social surplus
  - asymmetries between buyer and seller groups

Optimal Bayesian Mechanism

- $v_{(1)} > \dots > v_{(n)} > v_{(n+1)} \equiv \underline{v}$
- $c_{[1]} < \dots < c_{[m]} < c_{[m+1]} \equiv \overline{c}$
- **Optimal Bayesian mechanism** – maximizes expected revenue subject to IC and IR – trades q units, with q satisfying

$$\Phi(v_{(q)}) \geq \Gamma(c_{[q]}) \quad \text{and} \quad \Phi(v_{(q+1)}) < \Gamma(c_{[q+1]})$$

- DS implementation: buyers pay $\max\{v_{(q+1)}, \Phi^{-1}(\Gamma(c_{[q]}))\}$, sellers receive $\min\{c_{[q+1]}, \Gamma^{-1}(\Phi(v_{(q)}))\}$

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Empirical Virtual Types and Spacings

- Empirical distributions:

$$\hat{F}(j) \equiv \frac{n+1-j}{n+1} \quad \text{and} \quad \hat{G}(j) \equiv \frac{j}{m+1}$$

- Empirical virtual types:

$$\hat{\Phi}(j) \equiv v_{(j)} - \frac{1-\hat{F}(j)}{\frac{\hat{F}(j)-\hat{F}(j+1)}{v_{(j)}-v_{(j+1)}}} = v_{(j)} - j[v_{(j)} - v_{(j+1)}]$$

$$\hat{\Gamma}(j) \equiv c_{[j]} + \frac{\hat{G}(j)}{\frac{\hat{G}(j)-\hat{G}(j+1)}{c_{[j]}-c_{[j+1]}}} = c_{[j]} + j[c_{[j+1]} - c_{[j]}]$$

Simple Spacings-Based Mechanism

- Trade $k - 1$ units, where k is the largest number such that

$$\hat{\Phi}(k) \geq \hat{\Gamma}(k) \quad \text{and} \quad \hat{\Phi}(k+1) < \hat{\Gamma}(k+1)$$

- Buyers pay $v_{(k)}$, sellers receive $c_{[k]}$
- Issue: $\hat{\Phi}$ and $\hat{\Gamma}$ are highly volatile

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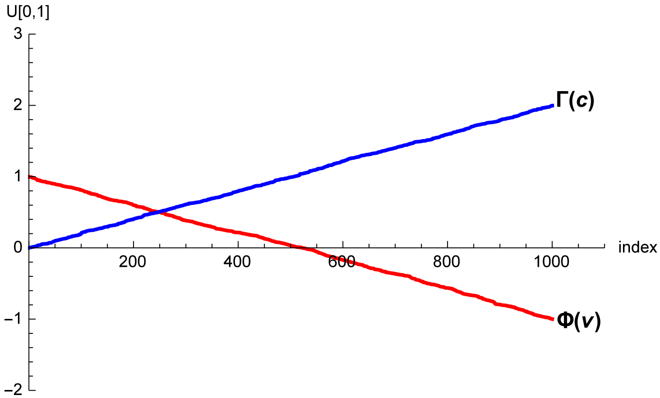
Theoretical and Empirical Virtual Types



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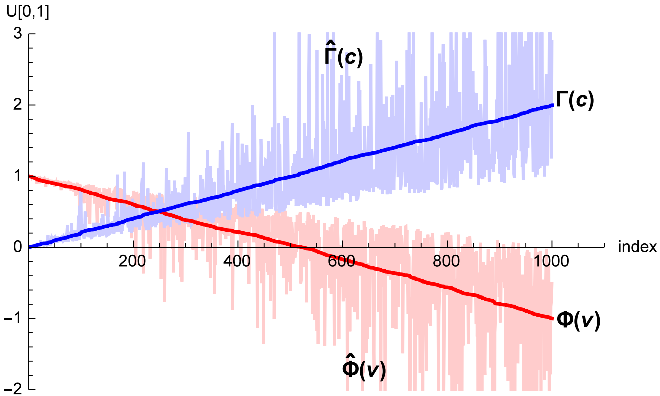

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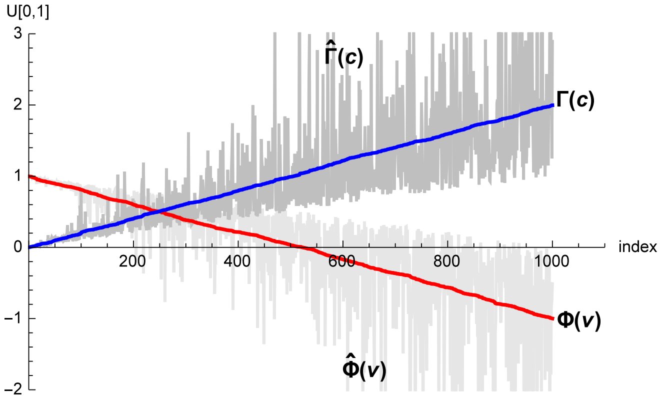

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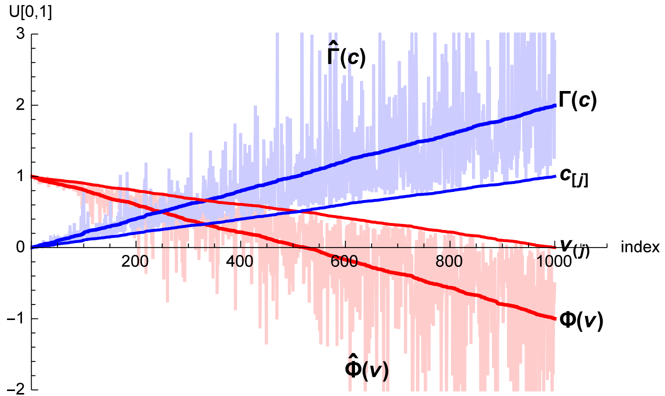

Smoothed Virtual Types

- Take the average of nearby spacings and reduce the coefficient to get **smoothed virtual types**:
- Smoothed virtual value

$$\tilde{\Phi}(j) \equiv v_{(j)} - (j-2)\frac{v_{(j)} - v_{(j+r_n)}}{r_n}$$

- Smoothed virtual cost

$$\tilde{\Gamma}(j) \equiv c_{[j]} + (j-2)\frac{c_{[j+r_m]} - c_{[j]}}{r_m}$$

- where r_n and r_m grow large with n and m, but at a slower rate (e.g. $r_n = \sqrt{n}$)

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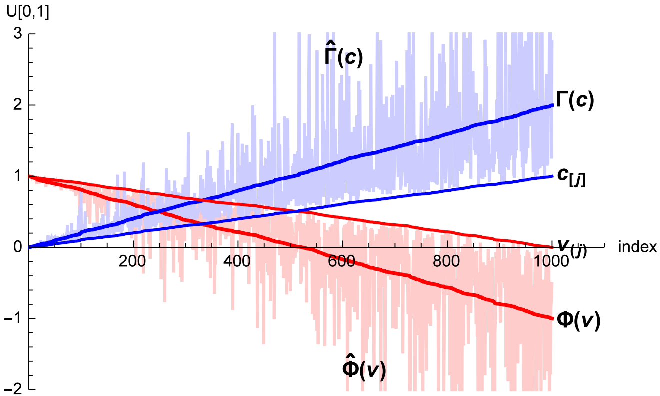
Theoretical, Empirical, and Smoothed Virtual Types



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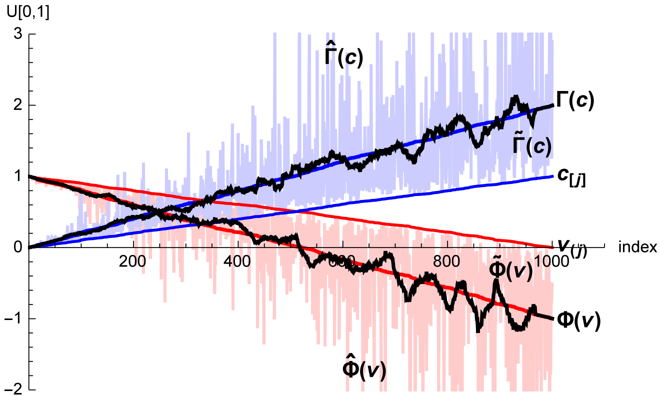

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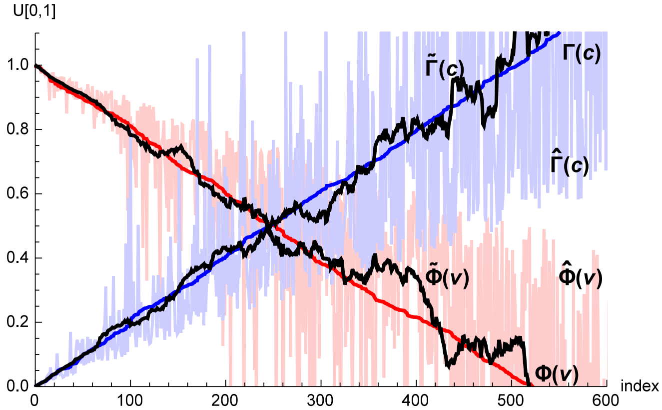

Properties of Smoothed Virtual Types

$$\tilde{\Phi}(j) = v_{(j)} - (j-2)\frac{v_{(j)} - v_{(j+r_n)}}{r_n}$$

$$\tilde{\Gamma}(j) = c_{[j]} + (j-2)\frac{c_{[j+r_m]} - c_{[j]}}{r_m}$$

1. For $j \geq 2$, $\tilde{\Phi}(j) \geq \tilde{\Gamma}(j)$ implies $v_{(j)} \geq c_{[j]}$
2. $\tilde{\Phi}(j)$ depends only on j and $v_{(j)}, .., v_{(n)}$
3. $\tilde{\Gamma}(j)$ depends only on j and $c_{[j]}, ..., c_{[m]}$

- 1 is important for deficit-freeness
- 2–3 are important for dominant strategies and non-bossiness
- non-bossiness is important for privacy preservation and clock implementation

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Baseline Prior-Free Mechanism

- Define $\tilde{\Phi}(n+1) = -\infty$ and $\tilde{\Gamma}(m+1) = \infty$
- Let $\tilde{k}$ be the largest integer s.t.

$$\tilde{\Phi}(\tilde{k}) \geq \tilde{\Gamma}(\tilde{k}) \quad \text{and} \quad \tilde{\Phi}(\tilde{k}+1) < \tilde{\Gamma}(\tilde{k}+1)$$

- Trade $\tilde{k} - 1$ units at prices $p_B = v_{(\tilde{k})}$ and $p_S = c_{[\tilde{k}]}$

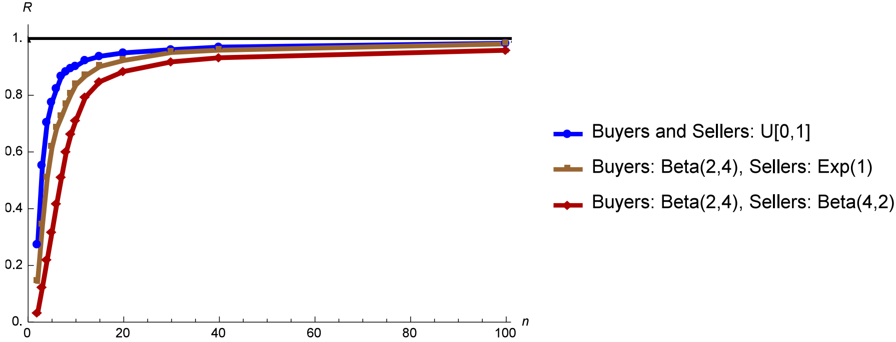
Distribution-Free Properties

- Dominant strategy incentive compatible
  - An agent who trades under truth telling cannot affect prices and still trade
  - An agent who does not trade under truth telling makes a loss when trading after a lie
- Ex post IR
- Envy free
- Non-bossy
- Deficit free
- Weak group strategy-proof

Asymptotic Optimality

- Compare prior-free to optimal revenue as $n \to \infty$ and $m \to \infty$
- A mechanism is **asymptotically optimal** if the ratio of the value of the objective – for now, revenue – under this mechanism over the value of the objective under the optimal mechanism converges in probability to 1
- **Proposition**: The baseline prior-free mechanism is asymptotically optimal.

Ratio of Prior-Free to Optimal Revenue



▸ Rate of Convergence

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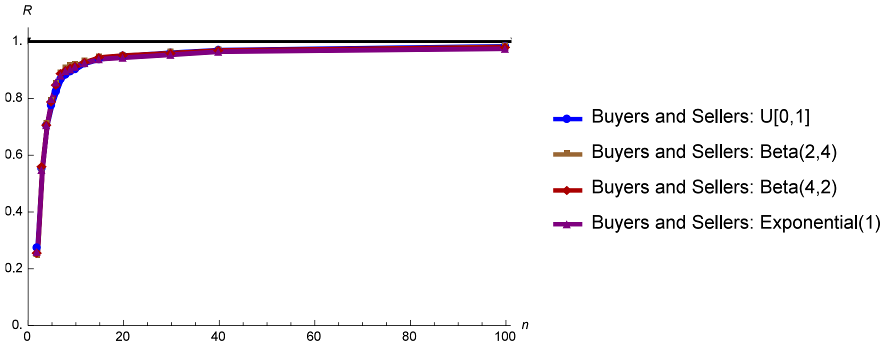

▸ Rate of Convergence

Asymptotic Optimality: Sketch of Proof

- Step 1: uniform bounds exist for the variance of the estimated spacings used in the smoothed virtual types (away from the boundary)
- Step 2: $\tilde{\Phi} - \Phi$ and $\tilde{\Gamma} - \Gamma$ are uniformly convergent in probability to zero (away from the boundary)
- Step 3: if $\underline{v} < \overline{c}$, the number of trades in the baseline prior-free mechanism approaches that in the optimal mechanism
  - Intuitively, if $\tilde{\Phi}$ and $\tilde{\Gamma}$ stay close to Φ and Γ, then the first intersection point of $\tilde{\Phi}$ and $\tilde{\Gamma}$ cannot be far from the intersection of Φ and Γ
- Step 4: the number of trades and payments converge in probability to the optimal level ▸ Proof details

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Generalization

- Extends to a more general designer objective:
  - α revenue $+ (1-\alpha)$ total surplus
  - Bayesian optimal mechanism is based on weighted virtual types:

$$\Phi_\alpha(v) \equiv v - \alpha \frac{1-F(v)}{f(v)} \text{ and } \Gamma_\alpha(c) \equiv c + \alpha \frac{G(c)}{g(c)}$$

    and trades q_α units iff $\Phi_\alpha(v_{(q_\alpha)}) \geq \Gamma_\alpha(c_{[q_\alpha]})$ and $\Phi_\alpha(v_{(q_\alpha+1)}) < \Gamma_\alpha(c_{[q_\alpha+1]})$
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Is this the best one can do?

- We define optimality in light of what is possible subject to privacy preservation
  - Privacy preservation matters (Hurwicz and Reiter 2006; McMillan 1994; FCC; Brandt and Sandholm 2005)
- Privacy preservation requires a clock implementation
- What does clock implementation require?

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Restrictions Imposed by Clock Implementation

Proposition

A direct mechanism can be implemented via a DCA if and only if it satisfies dominant strategies, non-bossiness, and envy-freeness.

- Show that DS, NB, EF imply the possibility of clock implementation
- DS: price faced does not depend on own report
- EF: all face same price
- NB: price can only depend on reports of nontrading agents
- DS and EF: buyers' price must increase and sellers' price decrease as the quantity traded decreases
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Privacy Preservation Comes at a Cost

Proposition

No α-optimal mechanism can be implemented as a DCA.

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- General impossibility result (any α and # of agents): Can't implement the ex post efficient or revenue maximizing (or any α-optimal) outcome in a privacy preserving way

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No α-optimal mechanism can be implemented as a DCA.

- Show the α-optimal mechanism violates NB
- In the α-optimal mechanism, buyer i trades iff $v_i \geq \max\left\{v_{(q_\alpha+1)}, \Phi_\alpha^{-1}\left(\Gamma_\alpha(c_{[q_\alpha]})\right)\right\}$, which depends on the report of the trading seller with type $c_{[q_\alpha]}$
- General impossibility result (any α and # of agents): Can't implement the ex post efficient or revenue maximizing (or any α-optimal) outcome in a privacy preserving way

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What is the best one can do s.t. privacy preservation?

- We define the **Bayesian Optimal Privacy Preserving** (BOPP) mechanism in terms of a DCA
- Increasing buyer clock p^B, decreasing seller clock p^S (if the number of active buyers and sellers differ, move one clock to induce exit)
- When the DCA ends, active agents trade at clock prices
- State: j active buyers and sellers remain
  - If $\Phi(p^B) \geq \Gamma(p^S)$, DCA ends
  - Otherwise, move clocks until exit or $\Phi(p^B) = \theta = \Gamma(p^S)$, where target θ is chosen optimally knowing F and G
  - If reach θ with no exit, DCA ends, otherwise continue

Optimal θ

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Prior-Free Approximation to the BOPP

- As in the BOPP, but use $\tilde{\Phi}$ and $\tilde{\Gamma}$ and estimate θ
- In the absence of estimation error, this **augmented prior-free** mechanism achieves the BOPP outcome

Illustration

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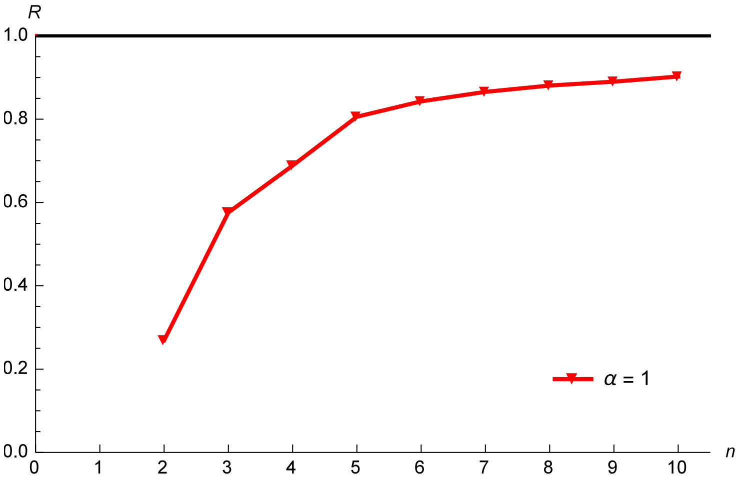
Illustration

- Ratio of prior-free to optimal outcomes (types drawn from the $U[0,1]$)



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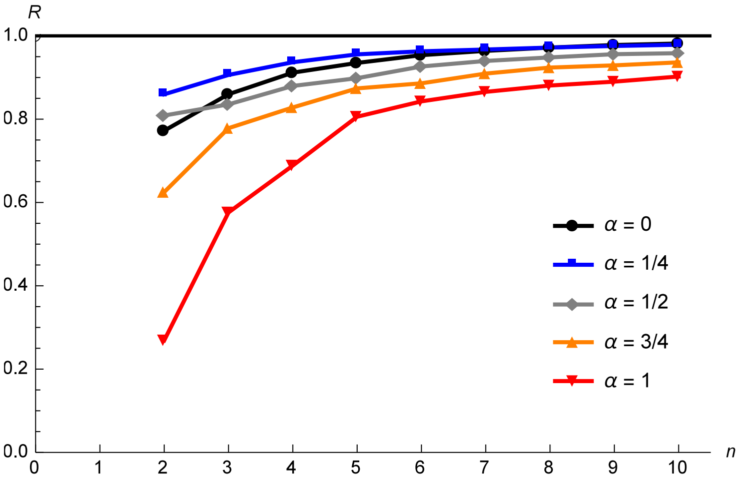

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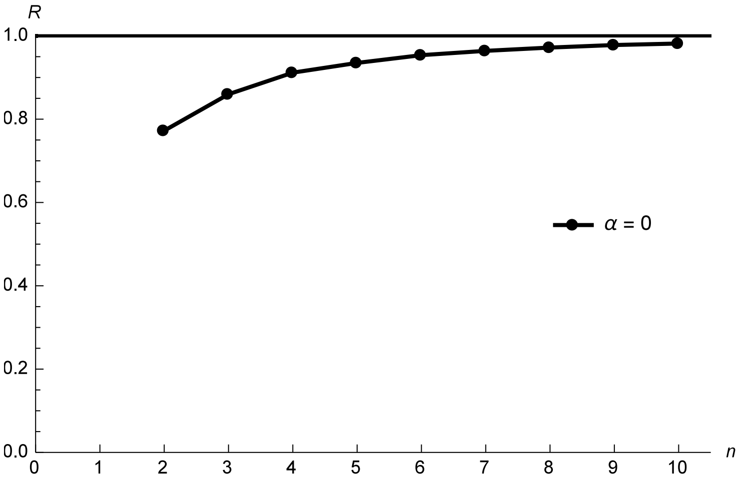

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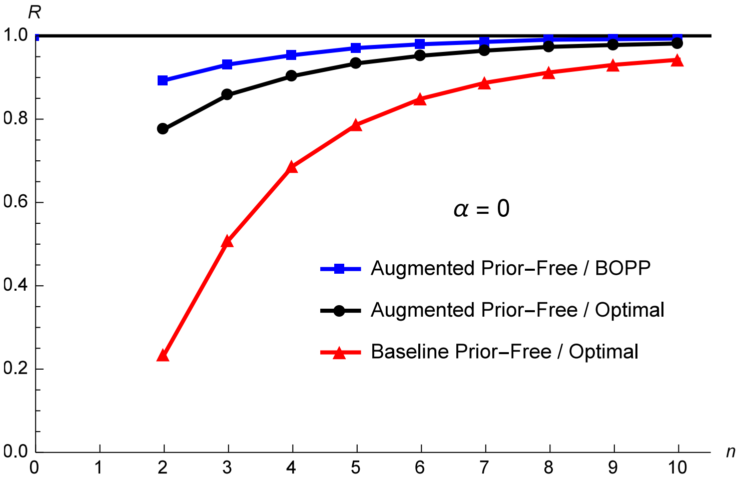

Performance in the Very Small ($n = m = 2$)

- Comparisons for $n = m = 2$ and $\alpha = 0$

	U[0,1]	Exp[1]	Beta[2,2]	B:Beta[2,4] S:Beta[4,2]
BOPP / Optimal	87%	79%	77%	60%
Augmented Prior–Free / BOPP	89%	99%	99%	92%
Augmented Prior–Free / Optimal	77%	78%	76%	55%
Baseline Prior–Free / Optimal	26%	26%	25%	3%

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Extension to Asymmetric Groups of Buyers and Sellers

- Buyers and sellers are divided into groups
- Buyers in group b draw from F^b and sellers in s from G^s
- Group membership is common knowledge
- Use a multiple-clock auction – synchronize buyer clocks to equalize virtual values across buyer groups, and similarly for sellers

Extensions: Alternative Objectives and Constraints

- Real-time diagnostics
  - Threshold α such that the DCA ends
  - Estimated change in social surplus from continuing
  - Estimated change in revenue from continuing
- Alternative objectives
  - Maximization subject to a revenue threshold
- Extensions to the multiple-clock auction
  - Caps on the number in a group that can trade
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7	0.28	0.87	−4.13	No		0.48	1.26	3.67
6	0.43	0.5	−0.46	No	0.12	−0.17	1.31	0.48
5	0.45	0.44	0.03	No	0.2	−0.22	0.84	0.53
4	0.49	0.35	0.56	No	0.39	−0.38	0.59	0.47
3	0.59	0.25	1.02	Yes	1.15	−0.48	−0.06	−0.16
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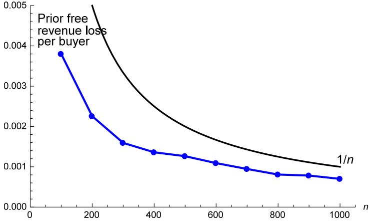
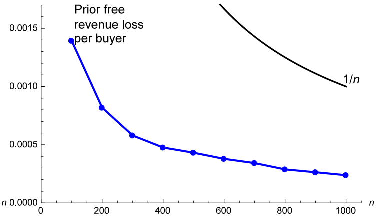
Conclusions

- We develop a two-sided mechanism that is prior free and permits an implementation via DCA
- Prior-free DCA:
  - Obviously dominant strategies
  - Privacy preserving
  - Asypmtotically optimal
  - Performs well in the small
- These properties provide robustness for practical problems, yet allow the mechanism to vary with relevant details, much like Bayesian optimal mechanisms do

- Spacings between order statistics (for well-behaved distributions) are on the order of $1/\min\{m,n\}$
- Suggests expected efficiency loss of that order



(a) B and S: Uniform(0,1)



(b) B: Beta(2,4), S: Beta(4,2)

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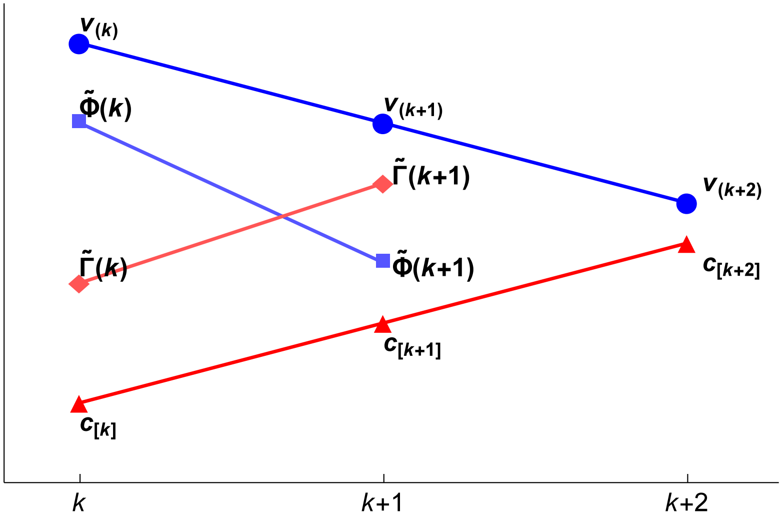

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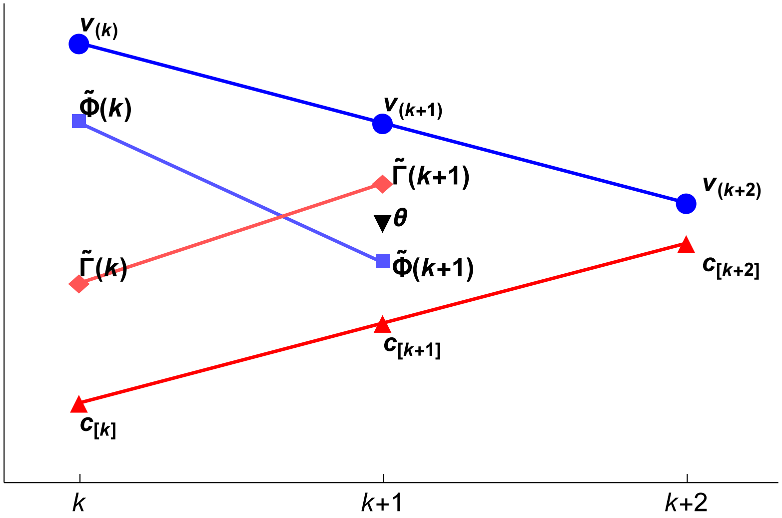
Illustration of Prior-Free Mechanism



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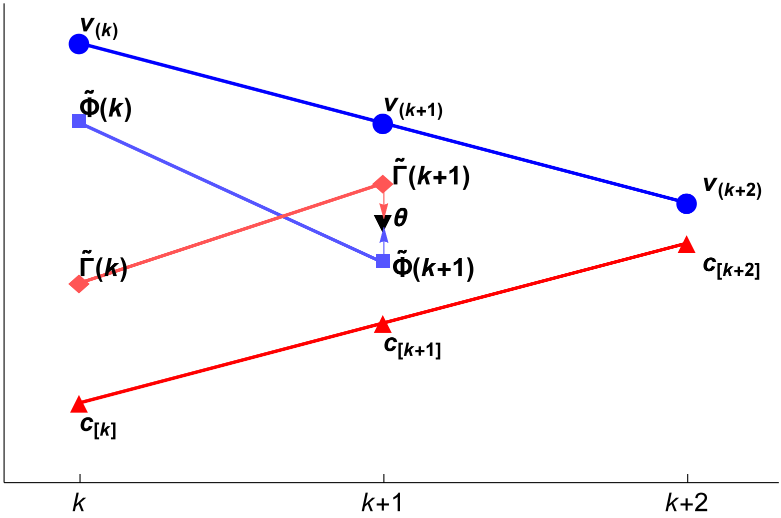

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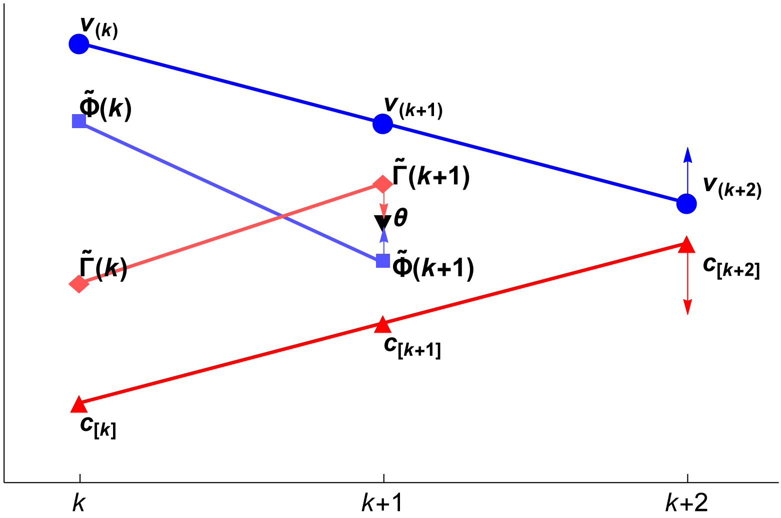

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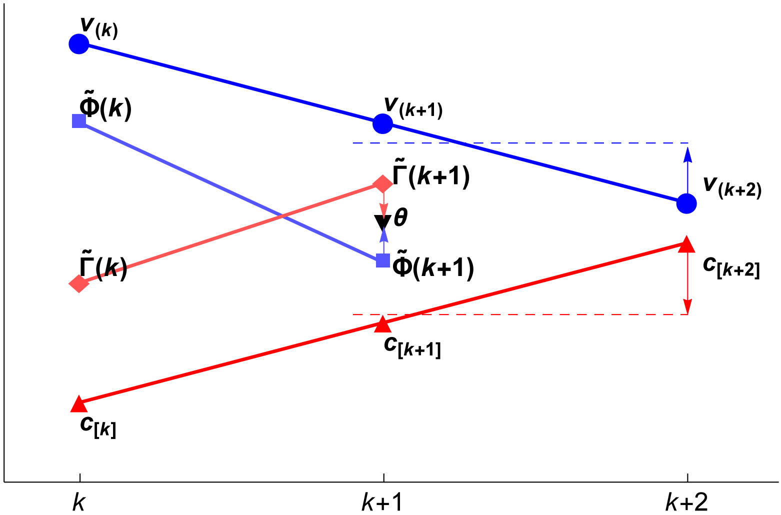
Illustration of Prior-Free Mechanism



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Proof: Asymptotic Optimality

- First note that: $plim_{m \to \infty} \Gamma(c_{[\rho m]}) - \tilde{\Gamma}(\rho m) = 0$
- Follows from the definitions and

$$E\left[\frac{G(c_{[j]})}{g(c_{[j]})}\right] = j\left(E\left[c_{[j+1]}\right] - E\left[c_{[j]}\right]\right)$$

Proof: Asymptotic Optimality

- Using $\frac{1}{r_m} \to 0$ and $\frac{r_m}{m} \to 0$, for $\rho \in (0,1)$,

$$\lim_{m\to\infty} Var\left[\Gamma(c_{[\rho m]}) - \tilde{\Gamma}(\rho m)\right] = 0$$

- Using this and Markov's inequality, for $\varepsilon > 0$,

$$\Pr\left(\left|\Gamma(c_{[\rho m]}) - \tilde{\Gamma}(\rho m)\right| \geq \varepsilon\right) \leq \frac{E\left[\left|\Gamma(c_{[\rho m]}) - \tilde{\Gamma}(\rho m)\right|^2\right]}{\varepsilon^2} \to 0$$

- Implies $p\lim_{m\to\infty} \Gamma(c_{[\rho m]}) - \tilde{\Gamma}(\rho m) = 0$
- Decreasing variance in ρ gives uniform convergence in probability

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Assuming differentiability of virtual types, any BOPP mechanism is characterized by θ_j^* such that

$$\frac{1 - F(\Phi_\alpha^{-1}(\theta_j^*))}{f(\Phi_\alpha^{-1}(\theta_j^*))} \frac{1}{\Phi_\alpha^{-1\prime}(\theta_j^*)} = \frac{G(\Gamma_\alpha^{-1}(\theta_j^*))}{g(\Gamma_\alpha^{-1}(\theta_j^*))} \frac{1}{\Gamma_\alpha^{-1\prime}(\theta_j^*)}, \qquad (1)$$

if such a $\theta_j^* \in [\Phi_\alpha(v_{(j+1)}), \Gamma_\alpha(c_{[j+1]})]$ exists, and otherwise if

$$\frac{1 - F(\Phi_\alpha^{-1}(\theta))}{f(\Phi_\alpha^{-1}(\theta))} \frac{1}{\Phi_\alpha^{-1\prime}(\theta)} < \frac{G(\Gamma_\alpha^{-1}(\theta))}{g(\Gamma_\alpha^{-1}(\theta))} \frac{1}{\Gamma_\alpha^{-1\prime}(\theta)}$$

for all $\theta \in [\Phi_\alpha(v_{(j+1)}), \Gamma_\alpha(c_{[j+1]})]$, then $\theta_j^* = \Gamma_\alpha(c_{[j+1]})$ and otherwise $\theta_j^* = \Phi_\alpha(v_{(j+1)})$.

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